

NATIONAL HIGHWAY TRAFFIC SAFETY ADMINISTRATION
WASHINGTON, D.C.

CONTRACTOR FINAL REPORT

TEST #

95

1/ 95 AFS

DOT HS-800 996

MODELING, SIMULATION AND VERIFICATION OF IMPACT DYNAMICS Volume 1, Executive Report

Contract No. DOT HS-031-2-481
February 1974
Final Report

PREPARED FOR
U S DEPARTMENT OF TRANSPORTATION
NATIONAL HIGHWAY TRAFFIC SAFETY ADMINISTRATION
WASHINGTON, D C 20590

Document is available to the public through
the National Technical Information Service,
Springfield, Virginia 22151

This document is disseminated under the sponsorship of the Department of Transportation in the interest of information exchange. The United States Government assumes no liability for its contents or use thereof.

1 Report No DOT HS-800 996		2 Government Accession No		3 Recipient's Catalog No	
4 Title and Subtitle Modeling, Simulation and Verification of Impact Dynamics - Vol. 1, Executive Report				5 Report Date 1 October 1973	
				6 Performing Organization Code	
7 Author(s) I. K. McIvor				8 Performing Organization Report No UM-HSRI-BI-73-4-1	
9 Performing Organization Name and Address Highway Safety Research Institute and Department of Applied Mechanics and Engineering Science The University of Michigan Ann Arbor, Michigan 48105				10 Work Unit No	
				11 Contract or Grant No DOT-HS-031-2-481	
12 Sponsoring Agency Name and Address National Highway Traffic Safety Administration U. S. Department of Transportation Nassif Building, 7th and E Streets, S. W. Washington, D. C. 20590				13 Type of Report and Period Covered Final Report June 28, 1972-Aug. 25, 1973	
				14 Sponsoring Agency Code	
15 Supplementary Notes					
16 Abstract This document is the summary report of an investigation of the Modeling, Simulation and Verification of Impact Dynamics. The goals of this study were (i) determine the state-of-the-art of computer simulation of vehicle impact relative to the needs of NHTSA, (ii) review the state-of-the-art of impact testing, (iii) assess the need and feasibility of extending the state-of-the-art, (iv) make recommendations for the development of advanced simulations. Detailed results of the study including the development of a component frame module for advanced simulations is discussed in the other three volumes of the final report. In this volume the results and conclusions of the entire study are summarized. In addition the report includes summary of current simulation capability, assessment of feasibility of developing advanced simulations, and a recommended methodology for their development.					
17 Key Words vehicle, crash, simulation, modeling			18 Distribution Statement Unlimited, available through the National Technical Information Service, Springfield, Virginia 22151		
19 Security Classif (of this report) Unclassified		20 Security Classif.(of this page) Unclassified		22 Price 56	

Form DOT F 1700.7 (8-69)

ACKNOWLEDGMENTS

The work under this contract was performed jointly by the Bioscience Division of Highway Safety Research Institute and the Department of Applied Mechanics and Engineering Science both of the University of Michigan. Participating staff members from HSRI were Drs. D. H. Robbins, H. C. Wang and B. Bowman. Faculty of AMES participating were Professors A. S. Wineman and W. H. Yang. A significant contribution to the project was also made by Professor W. J. Anderson of the Department of Aerospace Engineering.

We greatly acknowledge the cooperation of staff of the National Highway Traffic Safety Administration throughout the project. Discussions with Mr. Glen Brammeier and Mr. James Hofferberth were particularly helpful during our familiarization study. The interest and comments of Dr. Michael Chi, Mr. James Hackney, and Mr. John Tomassoni were appreciated. Finally the cooperation, criticism and advice of the Contract Technical Manager, Mr. Robert Krauss, were very helpful. His contribution is gratefully acknowledged.

TABLE OF CONTENTS

		Page
Chapter 1:	Introduction	
1.1	Goals of the Contract	1
1.2	Report Organization	3
1.3	Summary of NHTSA Simulation Needs	4
1.4	Definition of Simulation Spectrum	4
Chapter 2:	Summary of Conclusions	
2.1	Introduction	10
2.2	State-of-the-Art of Computer Simulation	10
2.3	State-of-the-Art of Impact Testing	11
2.4	Development of Advanced Simulations	13
2.5	Research Needs	14
Chapter 3:	Review of the State-of-the-Art of Computer Simulation	
3.1	Introduction	16
3.2	Current Modeling Concepts	16
3.3	Summary and Assessment of the State-of- the-Art	28
Chapter 4:	Assessment of Feasibility and Need for Future Development of Advanced Simulations	
4.1	Introduction	32
4.2	Need for Advanced Simulations	32
4.3	Technical Feasibility	33
4.4	Trade-Off Study: Computer Simulation Vs. Crash Testing	35
4.5	Economic Considerations	42
Chapter 5:	Recommendations for Development of Advanced Simulations	
5.1	Requirements for Advanced Simulations	45
5.2	Concept of Modular Development	46

TABLE OF CONTENTS
(Continued)

	Page
5.3 Definition of Required Modules	49
5.4 Research Needs	53
References	55

LIST OF FIGURES

	Page
Figure 1: Projected Savings for Use of Simulation	44

LIST OF TABLES

Table 1:	Selected NHTSA Functions and Associated Use of Computer Simulations	5
Table 2:	Simulation Spectrum	9
Table 3:	Summary of Current Simulation Programs	29
Table 4:	Vehicle Sub-Assemblies	48
Table 5:	Passenger Compartment Modules for Various Simulation Conditions	49
Table 6:	Requirements for Control Modules	50
Table 7:	Modules Required for Advanced Simulations	52

CHAPTER 1

INTRODUCTION

1.1 GOALS OF THE CONTRACT

The major mission of the National Highway Traffic Safety Administration (NHTSA) is to set reasonable and cost effective standards with respect to vehicle safety. Structural crashworthiness obviously plays a major role in this mission. The increasing concern with crashworthiness of automobiles has imposed the need for much greater understanding of vehicle structures in the crash environment. For this purpose the ability to model vehicle impact using computer simulation is attractive. The problem, however, is exceptionally complex and, in any general sense, beyond the scope of current technology. The purpose of this contract was to evaluate the capability of present technology and to assess the potential for further development.

Accomplishment of this purpose can be expressed in the following four specific goals of the contract:

- 1) Determine the state-of-the-art of computer simulation of vehicle impact relative to NHTSA needs.

In recent years a number of investigators have developed computer simulation programs to model the structural responses during vehicle impact. A variety of modeling concepts and degrees of sophistication have been employed. These programs have been systematically reviewed in order to assess current capability. Moreover for the purpose of this contract it is necessary to establish the simulation requirements of NHTSA and to relate the various programs to their specific needs.

- 2) Review the state-of-the-art of impact testing.

To date crash testing has played a dominant role in determining vehicle behavior during impact. Moreover it is essentially the sole method for establishing compliance with standards. Thus the capabilities of computer simulation relative to crash testing is an important factor in establishing the direction of future developments.

- 3) Assess the need and feasibility of extending the current state-of-the-art.

Recommendation to undertake further development of computer simulation requires demonstration that NHTSA has a clear need for advanced simulations. In addition, of course, such a recommendation must rest on a demonstration of technical and economic feasibility.

- 4) Prepare specific requirements for advanced simulation programs.

The final goal of the present contract is to provide a methodology for the development of advanced simulations. An integral part of this goal is to delineate specific areas of research essential to implementing the recommended methodology.

Finally a discussion of specific goals should mention the actual modeling study that was carried out under the contract. This was originally envisaged as a modeling cycle to provide input for an assessment of feasibility. As the investigation progressed, however, it became clear that the ability to develop general modules for the simulation of vehicle components was essential to the success of advanced vehicle simulations. Thus the modeling cycle was expanded into the

actual development of a general simulation module for the three dimensional, large deformation of vehicle frames.

1.2 REPORT ORGANIZATION

The results of the investigations conducted under this contract are presented in four volumes. They are:

Volume I Executive Report

Volume II State-of-the-Art: Computer
 Simulation of Vehicle Impact

Volume III State-of-the-Art: Impact Testing

Volume IV Three Dimensional Plastic Hinge
Frame Simulation Module

Volumes II and III are self contained discussions of the respective state-of-the-art reviews. Volume IV presents the theory, experimental verification study, and application of the frame module to an actual vehicle frame crush test. It also includes a user's guide and a complete listing of the current version of the simulation program.

In this volume, the Executive Report, the results and conclusions of the entire study are summarized relative to the goals of the contract. In the remaining sections of this chapter the simulation needs of NHTSA are summarized and a simulation spectrum is defined of sufficient breadth to cover the needs. This provides the necessary framework for stating the conclusions and recommendations of the study which are given in Chapter 2. A summary of current simulation capability is given in Chapter 3 which provides the background for the assessment of feasibility of developing advanced simulations which is discussed in Chapter 4. Finally Chapter 5 discusses the methodology for the development of advanced simulations.

1.3 SUMMARY OF NHTSA SIMULATION NEEDS

In carrying out its major mission, a number of functions are important to the planning and development of NHTSA's crash-worthiness effort. Computer simulation of vehicle impact provides a necessary research tool in support of these functions. A number of uses of computer simulation can be identified and correlated with specific functions. Such a study is summarized in Table 1.

It is clear that the required level of sophistication varies widely for the various simulation uses identified in Table 1. For example, the model used for a parameter study to ascertain the effect of mass-stiffness ratio on vehicle compatibility need not have the capability of a simulation program for verification of compliance with standards. Thus an attempt to develop a single simulation program for all NHTSA functions is not only economically unwise but also could inhibit focusing on specific issues.

1.4 DEFINITION OF SIMULATION SPECTRUM

It is evident from above that the simulation needs of NHTSA require a range of simulation capability. The required sophistication of the various simulation applications is discussed in detail in Volume II. This study led to the definition of a simulation spectrum defined by five levels of increasing sophistication. They are:

Level 1 Simulation:

Level 1 simulations are models with up to five or six degrees of freedom, the variables representing displacements and possibly rotations of lumped masses. Typically the model involves 2-3 lumped masses and a few (less than ten) generalized resistances. Detailed geometry and material behavior is not modeled. Geometry and the generalized resistances

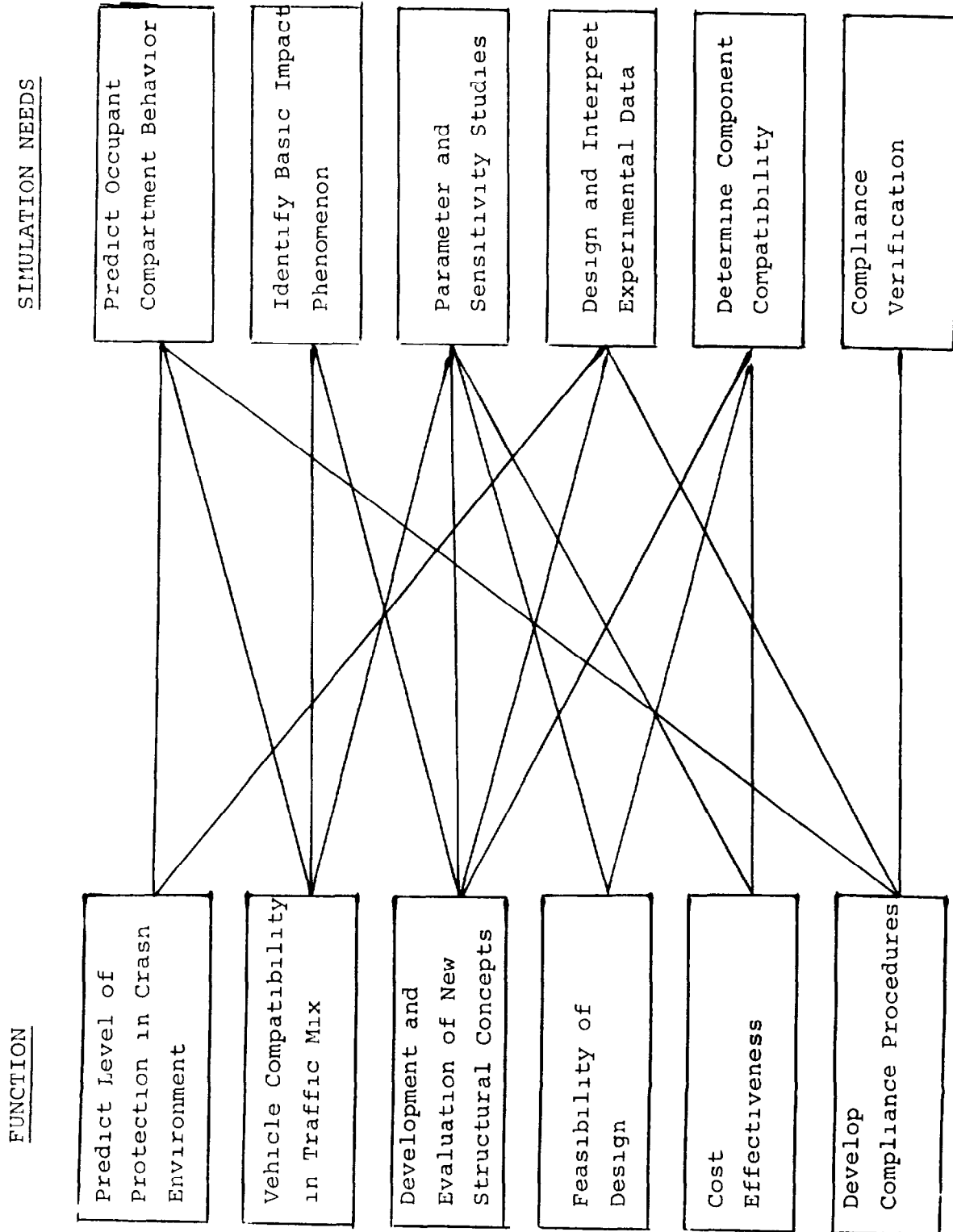


Table 1
Selected NHTSA Functions and Associated Use of
Computer Simulations

are defined by a small set of parameters. There is no attempt to relate the resistances to specific vehicle components, but rather they represent overall vehicle characteristics. The limited variables restrict results to overall gross displacements and average rigid body accelerations. The modeling is restricted to a specific loading situation. Level 1 simulation is designed for qualitative studies.

Level 2 Simulation:

Level 2 simulations are models with up to twenty degrees of freedom, the variables again representing displacements and rotations of lumped masses. The number of masses and generalized resistances may be greater than Level 1 simulation, but geometry and resistances are still defined by relatively few parameters. At this level, however, the generalized resistances represent specific vehicle components. The greater number of variables permit obtaining relative displacements between components. Generalized resistances are now related directly to force deformation characteristics of components, but the limited parameters permit modeling only the gross features. The modeling is restricted to a specific loading situation. Level 2 simulation is again qualitative but for a wider range of variables including the effect of specific components.

Level 3 Simulation:

Level 3 simulations also includes models with up to twenty degrees of freedom. The essential difference is the increase in sophistication in modeling component behavior. The force deformation behavior of the generalized resistances are obtained either from experimental tests or detailed static modeling of specific components. At this level the component tests or modeling will be for specific load conditions which restricts the simulation to similar loading

situations. Level 3 simulations give quantitative results which correlate with experimental data for relative displacements and average rigid body accelerations of the lumped masses. Generality of the results is restricted by the limited model variables.

Level 4 Simulation:

Level 4 simulations will have on the order of two to three hundred degrees of freedom. This level permits the dynamic modeling of major components including inertia and strain rate effects under reasonably general loading conditions. Other vehicle components will be modeled with less sophistication. The number of variables employed should permit sufficient detail to obtain displacement and acceleration time histories of a number of significant points in the vehicle including the occupant compartment for three dimensional motions. Level 4 simulations give accurate quantitative results for displacement and acceleration histories for the model variables employed.

Level 5 Simulation:

Level 5 simulation is a modeling of the vehicle structure in sufficient detail to give pointwise results for the displacement and acceleration histories throughout the vehicle. Probably in excess of one thousand degrees of freedom will be required. Modeling is based on material stress-strain behavior and detailed geometry of components. The modeling includes joint eccentricities, joint efficiency and local deformation effects. This level of simulation will give the displacement and acceleration environment of the occupant compartment in complete detail with accuracy of all variables within the confidence level of the input data.

Thus the simulation spectrum spans the range from simple qualitative models to general simulations capable of

predicting pointwise response. This spectrum is summarized in Table 2. Also indicated in the table are applications appropriate for the different levels. The introduction of this spectrum provides a measure for evaluating current modeling efforts and required future developments.

Simulation Level	Degrees of Freedom	Modeling Detail	Nature of Loading and Response	Confidence Level	Applications
1	10	Overall Vehicle Stiffness	Each Model Specialized for Particular Load, Gross Vehicle Deformation and Acceleration	Qualitative	Parameter and Sensitivity Studies
2	20	Limited Parameters for Gross Modeling of Component Force-Deformation	Model Specialized for Loading Relative Displacements for Limited Variables, Rigid Body Accelerations	Qualitative	Identify Basic Phenomenon, Parameter and Sensitivity Studies, Component Compatibility
3	20	Detailed Modeling of Major Components by Approximate Methods or Experiments	Model Specialized for Loading Relative Displacements for Limited Variables, Rigid Body Accelerations	Accurate Relative Displacements of Major Components and Average Rigid Body Accelerations	Predict Occupant Compartment Behavior, Identify Basic Phenomenon, Interpret Exp. Data, Component Compatibility
4	100-200	Accurate Modeling of Major Components Approximate Modeling of Sub-Components	General Loading Three-Dimensional Response-Limited Detail Occupant Compartment Acceleration	Accurate Displacements and Accelerations of Model Variables	Predict Occupant Compartment Behavior, Identify Basic Phenomenon, Interpret Exp. Data, Component Compatibility
5	Greater Than 1000	Detailed Modeling from Geometry and Material Behavior Including Joints and Local Effects	General Loading Pointwise Response	Accurate Displacement and Accelerations for All Points	Predict Occupant Compartment Behavior, Interpret Exp. Data, Compliance Verification

Table 2
Simulation Spectrum

CHAPTER 2

SUMMARY OF CONCLUSIONS

2.1 INTRODUCTION

The major conclusions and recommendations resulting from this study are summarized in the following sections. They are grouped in four main categories. In Section 2.2 the conclusions concerning the state-of-the-art of computer simulation are stated. A summary of the study leading to these conclusions is contained in Chapter 3 and full details are presented in Volume II of this report. The conclusions of the state-of-the-art study on impact testing are given in Section 2.3. The full study is reported in Volume III. Conclusions and recommendations concerning the development of advanced simulations are stated in Section 2.4. These results are based on the discussion contained in Chapters 4 and 5 of this report. Finally research needs required to support advanced simulations are listed in Section 2.5.

2.2 STATE-OF-THE-ART OF COMPUTER SIMULATION

Our conclusions concerning the state-of-the-art of computer simulation are:

- 1) Level 1 and Level 2 simulation needs of NHTSA are adequately met by available simulation programs. In particular, the BCL program is well designed to meet Level 2 simulation needs.
- 2) Within the restriction of collinear impact, Level 3 simulation may be obtained with hybrid models, i.e. models requiring experimental crush data for components as input data.

Although only limited application of the BCL model have been reported in this mode, it appears to serve as an adequate Level 3 simulation. Considerable care must be exercised in obtaining crush data in the appropriate dynamic deformation mode.

- 3) No currently available simulation based on a frame model has been qualified as a vehicle simulation. Moreover, it is unlikely that advanced simulations can be developed based solely on the frame concept. Nevertheless, both the frame program developed by Shieh based on the plastic hinge concept and finite element frame programs currently available have potential as "modules" for advanced simulations.
- 4) Although hybrid models adequately serve as Level 3 simulations, their potential for advanced simulations is small.

2.3 STATE-OF-THE-ART OF IMPACT TESTING

Conclusions reached based on the evaluation of the state-of-the-art of crashworthiness testing can be summarized as follows:

- 1) Techniques for retrieving structural crash response data from tests span the range of electromechanical motion, velocity, acceleration, and force transducers as well as optical recording techniques. In general a given technique is reliable. Comparison of data from different techniques is complicated, however, by differences in post-processing filters and by potentially unknown filtering inherent in the method.
- 2) Control of state variables such as impact velocity is highly important. High quality control is generally achieved in impact sleds and dynamic testing machines. Control has been a major problem in full scale crash tests, but current progress in the use of velocity gates and feedback control promises to minimize this problem.
- 3) The confidence level of a physical simulation technique as a realistic indicator of the crash event can only be estimated roughly at best. Two of the reasons for this are the variety of real world accident situations and the lack of criteria for comparison of data gathered in different tests.

- 4) Crashworthiness testing is expensive, usually costing a minimum of \$1000 for simple substructural component tests. Minimum costs for full scale test usually exceed \$4,000 and typically may range up to \$10,000.

Recommendations on the relationship of physical testing to mathematical crashworthiness modeling are summarized as follows:

- 1) Guidelines for verification experiments should be developed which define a realistic band of expected agreement between experimental results and model predictions based on the accuracy of model input data as well as filter properties of the systems producing both the experimental and the computer-generated data.
- 2) Crashworthiness model computer programs should include user-oriented preprocessor subprograms for aiding in the preparation of input data and post-processor sub-programs to present output in a form compatible with experimental data.
- 3) Techniques should be developed for estimating the overall properties (transfer function) of a system of filters in series to aid in quantifying the verifiability of a model before the fact and the level of agreement between model predictions and experimental results after the fact.
- 4) Research should be conducted to upgrade optical techniques for three-dimensional position measurement and the associated computer data-processing software.
- 5) Research should be initiated to develop new techniques of force measurement within structures.

In conducting this study the importance of filtering both experimental and computer-generated data became apparent and resulted in four additional recommendations.

- 1) Develop a catalog of specifications for analytical procedures such as integrating and differentiation as well as for all types of electronic and transducer hardware used in crashworthiness tests.

- 2) Because filters are in series in an analysis or an experiment, a filter system specification should be developed which includes the effects of all filters in the analysis or the experiment.
The functions of a specification of this type would be to: a. assign filtering limits to analytical and experimental procedures; and, b. ease the task of determining the possible level of agreement which should be expected between an experiment and a mathematical prediction.
- 3) Existing filter specifications such as SAE J211a should be updated and expanded to include the effects of phase shift and distortion.
- 4) Analytical techniques should be developed for waveform comparison in order to numerically define the degree of distortion, phase shift, and amplitude change. These procedures could be used in developing specifications of the accuracy which must be demonstrated by an analytical model in predicting a physical event.

2.4 DEVELOPMENT OF ADVANCED SIMULATIONS

Our conclusions concerning the need and feasibility of developing advanced simulations (Level 4 and Level 5) are:

- 1) Level 4 and Level 5 simulations would be of value to NHTSA in support of two major functions, prediction of the level of occupant protection and the design and implementation of compliance procedures.
- 2) The development of advanced simulations appear to be technically feasible. This conclusion is primarily based on three factors:
 - a) The potential of currently available frame programs for future development.
 - b) Our modeling study reported in Volume IV.
 - c) Preliminary indications from our modeling study that the critical area of joint behavior can be solved in a manner appropriate for advanced simulations.

- 3) The following conclusions were reached on the basis of a trade-off study between computer simulation and crash testing:
 - a) Advanced simulations cannot eliminate crash testing which is required to establish base lines with a high confidence level.
 - b) Qualified advanced simulations could reduce the required number of full scale tests significantly.
 - c) Flexibility of simulations makes them attractive alternatives to crash testing from most viewpoints. They are particularly useful for extrapolation and interpretation of results.
 - d) The potential reduction in the level of crash testing has substantial economic benefits.

On the basis of these conclusions we recommend that NHTSA support the development of advanced simulations. The most promising approach is a modular development that would provide a general program with the flexibility to optimize the model for a particular simulation. The basic modular concept and preliminary methodology is discussed in Chapter 5.

Our estimates for the cost and time for development of fully qualified advanced simulations are:

- 1) Level 4 simulation can be developed in two years at a cost of \$400,000-\$500,000.
- 2) Level 5 simulation can be developed in four-five years at a cost on the order of a million dollars.

It should be noted that potential economic benefits cited above were based on these development costs.

We recommend that Level 4 simulation be given priority in development for two reasons. First, some economic gain would be realized within two years, and second, many of the Level 4 modules will also be required for Level 5 simulations.

2.5 RESEARCH NEEDS

In addition to the development of modeling concepts for component modules, our study has indicated that a number of basic problems require

investigation in direct support of the development of advanced simulations. These research needs are discussed in some detail in Section 5.4 of Chapter 5. Here we list the areas in order of decreasing priority. They are:

- 1) Joint behavior in the large plastic deformation range.
- 2) Simplified but accurate models of local deformation.
- 3) Load-transmission characteristics of two dimensional structures.
- 4) Strain-rate sensitivity.
- 5) Numerical error control.

Chapter 3

REVIEW OF THE STATE-OF-THE-ART OF COMPUTER SIMULATION

3.1 INTRODUCTION

In recent years a number of investigators have developed computer simulation programs to specifically model the structural response during vehicle impact. A variety of modeling concepts have been employed to treat the large plastic deformations that are the essential feature of the vehicle impact problem. These programs are reviewed in detail in Volume II. By relating their capabilities to the simulation spectrum, the current state-of-the-art is established.

Since this study forms the basis for the assessment of the feasibility of extending the state-of-the-art, it is briefly reviewed in this chapter¹. The study also delineates the major difficulties that must be resolved in the development of advanced simulations. In the next section the various programs are individually discussed. This is followed in the final section by a summary and assessment of the current state-of-the-art.

3.2 CURRENT MODELING CONCEPTS

3.2.1 Simplified Spring-Mass Models

In general we define simplified models as those having two-three lumped masses and less than ten degrees of freedom. The masses are connected by generalized resistances which represent gross structural properties and are not specifically identified with particular vehicle components. There is a variety of such models in the published literature. Typical examples are given in references [2][3][4] which

¹This review is primarily a revised version of the survey paper [1].

serve as a basis of discussion here. The three models range from a single mass and spring to a three mass model with eight generalized resistances.

All three authors claim reasonable agreement between calculated results using the model and experimental results. It is noted, however, that agreement of displacement variables is considerably better than decelerations. Although peak deceleration may be quite close in the examples cited, the deceleration time curve is matched only in its gross features. This is more a function of judicious choice of parameters than a measure of model confidence. There is a high degree of arbitrariness in the definition of the generalized resistances employed in the model. All the authors employ piecewise linear force - deformation curves representing a plastic yielding structure. Each resistance represents a gross structural characteristic. For example, in reference [4] two of the resistances are defined as "front end upper member" and "front end lower member." The determination of the parameters characterizing the resistance is even more vague, as illustrated by a typical quote from reference [3], "The load - deformation characteristics of each nonlinear spring were determined by both presumptive calculations and experiments."

Thus, we conclude that agreement between model predictions and experiment represent a high degree of intuitive judgment by the investigator with a strong element of empirical curve fitting. This, of course, is not without merit. It demonstrates that simple models can describe qualitatively those features of vehicle impact which are compatible with the limited variables of the model. On the other hand a high level of confidence cannot be ascribed to quantitative results except for the experimental conditions (and possibly even more significant, the exact experimental procedure) to

which the model was "tuned." Thus simplified models are not useful as a predictive tool in a quantitative sense, but rather as a qualitative measure of general behavior.

With these limitations it is futile to pursue the question of the "best" model. Rather model selection should be based on choosing variables appropriate for the particular study. Vehicle parameters must be "tuned" by the investigator for the specific application based on experience and experiment. With this any number of simplified models will serve as Level 1 simulations. Typical examples of appropriate uses of such models for parameter and sensitivity studies may be found in the reports by Carter [5] and Spencer [6].

3.2.2 BCL Simulation Program

Battelle Columbus Laboratories (BCL) has developed a computer simulation program for colinear car/car and car/barrier collisions [7]. This program is based on a mathematical model with 4 masses and up to 35 individual nonlinear resistances. The masses are restricted to unidirectional motion.

Since the focus of BCL's study was to develop a flexible computer program, each mass or nonlinear resistance of the mathematical model does not represent any specific part or member of the vehicle. The determination of the candidate mass and resistance assignments are left to the user. He can leave these as blank, i.e. simplify the model, but cannot change the basic configuration. For a proper choice of masses and resistance, however, BCL's program can be applied to front, side and rear colinear impact.

In the program the characteristics of the resistance members can be classified into six different types, each being represented by a program subroutine. They are:

1. A model of elastic-plastic "spring" capable of

transmitting compression force only.

2. A model of a fixed-stroke variable-orifice hydraulic cylinder.
3. A model of an elastic-plastic "spring" which has both tension and compression capability.
4. A generalized model for elastic-plastic springs with tension and/or compression capability which may be described by a set of force versus deflection points and a representative unloading spring rate.
5. A model of variable-stroke, variable-orifice hydraulic cylinder.
6. A model of damping element which produce force proportional to velocity.

These various options for generalized resistances permit the representation of a wide variety of hypothetical force deformation relations. Thus with relatively simple input a broad range of component behavior can be modeled. With this capability the program meets all the requirements of a Level 2 simulation subject only to the restriction of a colinear impact.

Although to date only limited use of the BCL program as a predictive tool has been reported [8], it undoubtedly has the potential for Level 3 simulation. In this context, however, it is in the same category as the "hybrid" models discussed in the next sub-section. The basic difficulty is that there is no systematic way to determine the parameters of the hypothetical generalized resistances from the geometric and material properties of actual physical components. In principle they can be obtained from fitting the various options to experimental crush data. In fact, using option 4, experimentally determined curves can be used directly. In either case its use as a

predictive tool requires experimental crush data for each component. The limitations of such "hybrid" models are discussed below.

Finally it should be noted that the BCL program has an option to incorporate a dynamic correction factor to component force-deformation data. It has the form

$$F_{\text{dynamic}} = C_v \cdot F_{\text{static}}$$

where

$$C_v = A + B \log_{10} V_o$$

in which V_o is the impact velocity. A and B are chosen to give $C_v = 1.3$ at an impact velocity of 30 MPH. Such an overall magnification factor must be considered empirical and should be used with caution. This point is discussed in more detail in the next section.

3.2.3 Hybrid Simulation²

At the present time, hybrid simulation based on the work of Kamal [9] has been the most successful approach to predictive capability for vehicle impact. Its use has wide acceptance within the automotive industry. To our knowledge there are two operating programs in use, the Kamal program at General Motors and the CSS program employed by Autosafety Engineering Corporation [10]. Both are considered proprietary in detail, but their general features and application to specific problems are available.

The present programs are basically three lumped masses with eight resistances. The resistances are identified with specific vehicle components or subassemblies. The force deformation curve for each resistance is determined experimentally from static crush tests and supplied to the program in digitized tabular form. Dynamic resistances are accounted

²We use the term "hybrid" to denote simulations requiring experimental crush data for components as program input.

for by an empirical "strain rate factor." The programs are limited to colinear front or rear impact.

The demonstrated results for frontal impact are good. Accurate values for the relative deformation of components and overall vehicle crush are obtained. The energy dissipated in each component is also obtained and the total energy accounted for within a few percent. The computed rigid body accelerations are less satisfactory but sufficient to make engineering judgment on design. Typically experimental results for accelerations show high frequency oscillations about an average value. The high frequency peaks are not obtained in simulation, but the average value is predicted with engineering accuracy.

In evaluating the present programs there are two major problems that limit their general use. The first is the dynamic correction factor. Although there is considerable information on dynamic stress-strain curves for common metallic materials, equivalent information for structural force deformation curves is not known. The basic difficulty is that the strain rate may vary spatially over the structure with local strain rates differing by order of magnitude from the average rate. Thus, at the present time the dynamic factor is set empirically. This requires considerable judgment and experience. There is evidence that different factors may be required for different structural configurations.

The second problem is the care that must be exercised in conducting the static crush tests. Correct simulation depends upon the static deformation mode coinciding with the dynamic mode. The crush test must be carried out to insure this similarity. This may require special constraints and/or

loading procedures. Again considerable judgment and experience must be exercised in the design of the tests. These problems in general reduce the confidence level of the simulation in the absence of experimental confirmation for a particular run. This is due to the difficulty of objectively measuring the judgment factors involved and reliance must be placed on subjective evaluation of the experience of the investigator.

There are also some difficult problems in generalizing the present simulations to other crash environments. Even a relatively simple situation as an unsymmetric pole test presents major difficulties. The crucial problem is to define the experimental information required which is consistent for a given model. When the only degrees of freedom are unidirectional translational displacements, the required force-deformation curve is relatively easy to define. When other displacement and rotational degrees of freedom are introduced, which is necessary for any type of unsymmetric loading, the problem is much more difficult. For the large plastic deformations of interest, the force and moments transmitted to the lumped mass will depend upon all the degree of freedom variables. How to define a series of tests to experimentally determine this function of several variables is not obvious. Further the correlation between analytically defined degrees of freedom and physical measurements is difficult in the three dimensional situation. Finally insuring the appropriate deformation mode presents additional difficulties.

We conclude that currently used hybrid models provide Level 3 simulation capability within the restriction of colinear impact. Their use, however, requires experience and judgment in obtaining appropriate experimental crush data. Finally the potential for generalizing hybrid models to higher level simulations is small.

3.2.4 Frame Models

Recently a number of investigators have independently developed more general programs directed towards Level 4 and Level 5 simulations. Although a variety of structural techniques have been employed, they all model the vehicle as an assemblage of frame members interconnected at discrete nodes. The frame members are taken as straight beams with uniform cross section between nodes. Inertial modeling consists of lumped point or rigid body masses at the nodes. With one exception the simulations are three dimensional and allow for general loading conditions. In the following paragraphs we briefly review these programs.

The first approach is the dynamic elastic-plastic response of planar frames presented by Shieh [11]. The basic simplifying assumption is the structural concept of a plastic hinge. The analysis permits large changes in structural geometry, but assumes that plastic deformation occurs only at the nodes. The deformation between nodes is taken as elastic and hence is assumed small. The location of all potential plastic hinges must be specified a priori. The method of assigning lumped masses at the nodes is left to the judgment of the user.

A number of approximations and assumptions are inherent in introducing the concept of a plastic hinge. In addition to assuming the extent of the plastic zone is small, it also neglects any elastic-plastic bending at the cross section. Thus the cross section is considered either fully elastic or fully plastic as determined by the yield condition. In the present study the effect of axial force on the yield condition is neglected. Thus a hinge is introduced whenever the bending moment at the node reaches a critical specified value. The moment is then specified to be constant until the rate of

plastic work becomes relevant and is again considered to be elastic.

The simulation is reasonably accurate under these assumptions. Correlation with experimental results has been demonstrated for specialized frame structures. Correlation to overall deformation and average acceleration for a vehicle subjected to impact correlation has not been demonstrated.

The results obtained, however, have demonstrated the usefulness of the plastic hinge formulation for preliminary thinness studies. The current restriction to planar loading, of course, limits its use as an overall vehicle simulation. Even for symmetric loadings, biaxial bending and torsion will be induced in typical automotive frame structures. It should also be noted that the assumptions inherent in the concept are too restrictive for predicting the detailed response associated with Level 5 simulation. This follows from the fact that realistic relationships between the stress-strain resultants and the deformation cannot be established without detailed consideration of the stress distribution on the cross section. As established in Chapter 5, however, there is a need for a cost-effective Level 4 simulation. The Shieh program has considerable potential for this purpose.

Recently a different approach has been employed by Wittlin and Gamon [12] in their simulation program "KRASH." This program was developed for aircraft type structures. In principle, however, it is applicable to vehicle impact. In concept it is a three-dimensional extension of the BCL model consisting of masses connected by straight line one-dimensional "beam" elements. Each mass now has six degrees of freedom, three translational and three rotational. The model equations are obtained by writing the equations of motion for each mass by summing the forces and moments acting on the mass from the

generalized beam resistances. The program includes occupant masses that may be coupled to the structure.

In treating the generalized resistances, however, the program is essentially a frame model. Each "beam" element transfers a general force (three components) and general moment (three components). Thus the structure is replaced by an equivalent three-dimensional frame. The large deformation is treated by piecewise linearization. In each time step the forces and moments are determined from a linear stiffness matrix (the elastic stiffness matrix) which is adjusted for plasticity by multiplying by a stiffness reduction factor. The stiffness reduction factor is experimentally determined from overall force (moment) - displacement (rotation) curves obtained from static crush data. In this respect it is a generalization of the "Kamal" model.

Although the KRASH program appears to have potential as a general three-dimensional Level 3 simulation, there are serious questions about the feasibility of the procedure. The stiffness reduction factor concept employed in the program is theoretically incorrect in three-dimensional problems. The procedure employed implies that each element of the plastic stiffness matrix depends upon the current value of only a single deformation variable, whereas in general they depend upon the entire deformation history. Thus it is impossible to define a unique "load-stroke" curve for the experimental determination of the reduction factor as postulated by the KRASH formulation.

We conclude that experimentally determined stiffness reduction factors are meaningful only if the component test closely duplicates the dynamic deformation experienced in the actual vehicle impact. It is questionable whether this is experimentally feasible for general three-dimensional response

except possibly under very special loading conditions. In addition the experimental difficulties discussed above in connection with extending the Kamal model are relevant here.

Thus it is likely that KRASH can be used as a Level 3 simulation only under restricted circumstances. It may prove useful as a three-dimensional Level 2 simulation where hypothetical reduction factors can be chosen based on experience and judgment for a particular qualitative study.

A more general finite element frame model has been developed by Young [13] in the simulation program CRASH. The program is three-dimensional and considers both geometric and material nonlinearities. Material behavior is limited to plasticity theory. The basic beam element has uniform properties, but nodes may be specified arbitrarily. No prior assumption on location of plastic zones is required. Inertial modeling is accomplished by lumped masses at the nodes, the assignment of masses being left to the judgment of the user. Moments and forces at the nodes are computed by numerical integration of the stress distribution over the cross section. Thus the actual stress-strain behavior of the material may be used directly at the expense of monitoring the stress state at locations across the cross section.

The formulation of the CRASH simulation is analytically sound and does not rely on simplifying assumptions in its treatment of plasticity. Its present applicability to general vehicle simulation, however, is questionable. The CRASH program has been used by Melosh [14] to model the vehicle to barrier impact of a Mustang. The simulation was not successful. The model was much too stiff. Passenger compartment acceleration peaks occurred earlier and were of higher duration than the test results.

It is unlikely that these results could be improved significantly by using more elements. The basic difficulty is the inadequacy of the frame concept to model the entire vehicle. At the present time there is no rational way to choose cross sectional properties so that a beam is equivalent to many actual structural components. Another source of modeling error is the structural joints. In the Melosh simulation the joints are treated as frame nodes which essentially neglects any effect of joint inefficiency. Also local deformation of the cross section is not considered.

The final frame program to be discussed has recently been developed by Thompson [15]. The program is proprietary, but a general description is given in the reference cited. Basically the program is a finite element frame program with nonlinear geometry and plastic deformation capability. Although differing in some key respects, it is similar in size and concept to CRASH. It is considerably more flexible in treating cross sectional properties and is thus more adaptable to vehicle modeling. (As with all frame models, of course, the basic modeling problem of replacing actual components with equivalent beams remains.) It is also more general in material properties including strain rate sensitivity.

It also differs in another important respect. Rather than derive a plastic stiffness matrix which must be recomputed at each time step, the program employs an elastic stiffness matrix and a stiffness reduction factor. Unlike KRASH, where the reduction factor is postulated as being known from experiment, the present program computes this factor at each time step by taking the ratio of the actual moment about the neutral axis to the fully elastic moment. This requires pointwise integration across the cross section and an iterative procedure for converging to the plastic

stress-strain curve at each point. This is computationally a major task. Relative efficiency between this and the CRASH formulation is not known, but they are probably computationally of the same order of magnitude.

Although the Thompson reduction factor accounts for deformation history, it still may be criticized on theoretical grounds. The procedure is valid for symmetric bending, but in general is not correct. The range of loading conditions for which the procedure will give reasonable results is speculative. We believe, however, that reasonable results can be expected provided the resultant moment vector has small deviation from the neutral axis and torsion and axial effects are not significant.

In reference [15] correlation between results of simulation and tests was demonstrated for two experiments. The first was a dynamically loaded beam, and the second was a side impact study. In both cases the program was used to predict the time-varying nodal forces when the experimental nodal displacements were used as input at each time step. This is quite different, of course, than predicting the dynamic response from initial conditions. Thus on the basis of published results, the Thompson model cannot be considered as fully validated.

3.3 SUMMARY AND ASSESSMENT OF THE STATE-OF-THE-ART

In the preceding section we have discussed the capabilities and limitations of currently available simulation models. The discussion is summarized in Table 3. An assessment of the current state-of-the-art based on this summary leads to the following conclusions:

1. Level 1 and Level 2 simulation needs of NHTSA are adequately met by available simulation programs.

Program	Type	Qualified Simulation Level	Restrictions	Potential Simulation Level	Needed Development
Simplified Models	Spring-Mass	Level 1	Specialized Conditions, Qualitative	--	--
BCL	Spring - Mass, General Configuration	Level 2 Level 3	Colinear Qualitative, Limited Verification Exp. Crush Data	--	--
KAMAL	Spring - Mass	Level 3	Colinear Experimental Crush Data	--	--
KRASH	3D Frame Experimental Stiffness Reduction	Level 2	Limited Validity of Reduction Factor, Qualitative	--	--
SHIELH	Planar Frame Plastic Hinge	--	Plane Motion, Ideal Plastic Hinge	Level 4 Module	Generalization to 3D
CRASH	3D Frame Plastic Finite Elements	--	Ideal Frame Elements	Level 5 Module	Account for Local Deformation and Joint Behavior
THOMPSON	3D Frame Finite Element with Reduction Factor	--	Ideal Frame Elements, Reduction Factor	Level 5 Module	Local Deformation and Joint Behavior, Generalize Reduction Factor

Table 3

Summary of Current Simulation Programs

2. Within the restriction of collinear impact simulation may be obtained with hybrid models. Experimental crush data for components in the appropriate dynamic deformation mode is required.
3. No currently available simulation based on a frame model has been qualified as a vehicle simulation.

The most striking feature of the current state-of-the-art is the success of hybrid models for quantitative prediction. When taken together there are no published reports of qualified vehicle simulations using the more analytically sophisticated frame models. There are two factors that account for this situation. Despite their apparent greater modeling detail, no current frame simulation accounts for local deformation of the cross section. Further joint efficiencies³ and eccentricities are not taken into account. Both effects play a significant role in the energy dissipated by the structure and are inherently accounted for in experimental crush data. The second factor is that the single force deformation curve required for collinear impact can be obtained experimentally for non-frame components like exterior sheet metal, fire wall, unitized forestructure, motor mounts, etc. In contrast there is no rational way to choose the cross section properties of an equivalent beam element to use in a frame model. Thus the evidence strongly suggests that a purely frame model is inadequate for a complete vehicle simulation. In addition advanced simulations cannot be realized without including effects of local deformation and joint behavior.

The current computational success of hybrid models have, however, about reached their maximum potential as an overall vehicle simulation. It is unlikely that they can be developed beyond their present Level 3 simulation capability. The major technical difficulty is the problem of obtaining the required experimental relationships between the generalized forces and generalized displacements for three dimensional deformations.

³The Thompson model incorporates an empirical joint efficiency factor but the choice and use of this factor was not discussed.

For collinear impact only a single force and displacement variable are involved. In the general case, however, not only must a matrix relation be determined, but also this relationship is not unique and depends upon the loading history. Thus a definitive experiment cannot be performed. This greatly limits the hybrid concept since three dimensional crush data must be obtained, which in itself is a major task, for every loading configuration.

In contrast the frame simulation programs have demonstrated considerable potential for advancing the state-of-the-art. As discussed above frame models are also inadequate for overall vehicle simulation. They can serve as accurate modeling techniques for major vehicle components and thus serve as the basis for advanced simulations.

With respect to the potential of specific simulations, the frame program KRASH has major deficiencies. The empirical stiffness reduction factor makes KRASH a three dimensional version of the hybrid concept. The major experimental difficulties probably precludes its use except for qualitative studies. The current Shieh program is also limited due to its restriction to planer frames. It does, however, have merit for use as a module in Level 4 simulation if it is generalized to three dimensional deformation. The program CRASH and the finite element program of Thompson both consider the detailed elastic-plastic stress distribution over the cross section. This computational complexity precludes their use for Level 4 simulation, but probably will be required in a Level 5 frame module.

ASSESSMENT OF FEASIBILITY AND NEED FOR FUTURE DEVELOPMENT OF ADVANCED SIMULATION

4.1 INTRODUCTION

The state-of-the-art study has shown that the current simulation capability is currently available. Although it is desirable to reduce the dependence on expensive crash testing, this is an issue in long term development plans. The study is concentrated on the question of development of advanced simulation capability at Level 5 capability.

There are a number of factors that need to be considered in addressing this question. To recommend NHTSA support for the development of advanced simulations, we must demonstrate (i) a need relative to NHTSA functions, (ii) technical feasibility, (iii) a favorable trade-off with other methods of meeting NHTSA needs, and (iv) economic feasibility. These factors are discussed in the following sections.

4.2 NEED FOR ADVANCED SIMULATION

In the Familiarization Study (contained in Volume I of this report), we identified two major functions of NHTSA in which advanced simulations could be employed. These two functions are the ability to predict the level of occupant protection in general crash situations and the design and implementation of compliance procedures. These areas are of crucial importance. The formulation of compliance procedures is an integral part of the rule making effort. Occupant protection, of course, is the basic goal of all crashworthiness efforts.

The question of supporting these functions with alternatives to computer simulation, primarily crash testing, is discussed in Section 4.4. Here we focus on the need for additional development if these functions are to be supported by computer simulation. With respect to occupant protection, current simulation capability is limited to predicting average acceleration data for the passenger compartment under limited loading conditions. Thus at the present time prediction of occupant protection must be based on

highly simplified injury criteria.

To date this has not imposed severe restrictions since our knowledge of the relation between the environment experienced by the occupant and actual injury has been extremely limited in a quantitative sense. This situation is rapidly changing. Both within NHTSA and in other governmental and private organizations major efforts in biomechanics are underway to develop occupant models which will permit relating detailed injury mechanisms to occupant loading. Current simulation of vehicle impact is not compatible with these efforts in the sense it will not be able to provide the detailed occupant loading required to determine injury. Thus if simulation is to provide a predictive tool in crashworthiness studies of occupant protection, advanced simulations will be required in the near future.

With respect to compliance procedures, there is no question about the inadequacy of current simulation capability for this purpose. To date, however, there appears to be general satisfaction with the use of crash testing for judging compliance with standards. But this situation can also be expected to change. With increasing knowledge of the biomechanical behavior of occupants, future standards are likely to be stated in terms of detailed occupant injury. Design of compliance procedures will require relating structural response to injury mechanisms. Only advanced computer simulation can support this effort.

4.3 TECHNICAL FEASIBILITY

Level 4 and Level 5 simulation capability is beyond the current state-of-the-art. Thus the development of advanced simulations requires more than the codification of present knowledge. Nevertheless substantial progress has been made in modeling of vehicle impact, and there is sufficient evidence on which to base a judgment of technical feasibility.

As will be discussed in detail in Chapter 5 the requirements for advanced simulations can best be met by the development of vehicle component modules that can be automatically assembled under user control for specific overall simulations. Within the current state-of-the-art we already have developed a good foundation for modeling major vehicle components. The

current frame simulation programs, although unsuitable for overall simulation, have demonstrated that finite element methods can model large plastic deformations of vehicle type frames to a degree suitable for Level 5 simulation. There remain basic questions about joint behavior, but the technical applicability of the method has been demonstrated. On the other hand, lumped mass-generalized resistance models also have potential as a modeling concept for specific mechanical sub-assembly modules.

It is clear, however, that other modeling concepts need to be developed. Exclusive reliance on methods employed in the finite element frame programs for all modules of deformable structural components will be highly inefficient and probably economically prohibitive. The success of the modular concept depends upon a variety of modeling techniques to treat each component with the minimum sophistication required for a given type of overall simulation. Moreover it is essential that Level 4 simulation capability be developed, since the exclusive employment of Level 5 simulations for all potential uses of advanced simulation represents "overkill" and again would raise serious economic questions.

The modeling study [16] performed under this contract has demonstrated the technical feasibility of developing computer codes whose size, accuracy, and formulation are appropriate for a general Level 4 simulation module. The model is based on extending the concept of a plastic hinge to the three dimensional deformation of frames, but with a formulation that has all the flexibility and generality of application usually associated with finite element programs. Some basic questions still remain, primarily the question of joint behavior. Also the extension of this concept to two dimensional structures is an open question of considerable importance. Nevertheless, we feel the study has demonstrated the basic feasibility of Level 4 simulations.

Another aspect of the development of advanced simulations is the need for adequate methods of numerical computation and hardware capability. The current state of numerical methods relative to vehicle simulation is reviewed in Volume II of this report. Algorithms for solution of large

system of equations and for forward numerical integration are now well developed. A number of applications of these methods to structural problems which in size are comparable to advanced vehicle simulations are discussed in reference [17]. For the vehicle problem a number of questions remain, particularly the definition of error measures and their use to control forward step size. These are, however, primarily questions of efficiency necessary to optimize programs for production use and do not represent fundamental problems. We conclude that numerical analysis and computer capability is adequate for developing advanced simulations.

Although the above discussion indicates that a substantial base exists for the development of advanced simulations, it has also identified some basic research questions. Probably the most crucial area is the question of joint behavior in the large plastic deformation range. For advanced simulations to be successful, it will be necessary to accurately model joint behavior without employing a three dimensional finite element analysis. Some preliminary progress on this problem has been made. N1 [18] has shown that the local deformation of a box beam could be accounted for by a correction factor to the moment - curvature relation: Our modeling study [16] also showed the possibility of defining a functional relation analogous to the yield function which incorporates observed behavior in an empirical way. Thus there is some evidence that the solution of this problem is feasible in the near future.

4.4 TRADE-OFF STUDY: COMPUTER SIMULATION VS. CRASH TESTING

In the area of structural crashworthiness the question of computer simulation vs. crash testing is complex for a number of reasons. Both are used for a variety of purposes, and their relative merits may differ with function. We wish to employ both methods as a means of ascertaining behavior in the real world environment. This adds an additional dimension to the comparison. Finally it is important to remember that the two approaches are not mutually exclusive. In addition to the obvious need for experimental qualification of simulation models, the complexity of the vehicle impact problem and the variability

of behavior between nominally similar vehicles predicted by use of a single tool.

Thus the purpose of the study is not to establish the merit of one approach compared to the other, but to delineate the relative strengths and weaknesses in a given situation. In particular, computer simulation have sufficient advantages relative to crash testing for the functions of importance to NHTSA to justify the development of advanced simulations.

In what follows we are primarily concerned with the comparison between full scale vehicle crash tests and overall vehicle simulation. As discussed above, we will focus specifically on two major functions of NHTSA, the ability to predict the level of occupant protection in general crash situations and compliance verification. At the present time quantitative information is available in these areas only from crash testing. On the other hand most of the computer simulation needs required by the other functions identified in this study are essentially met by the Level 2 and Level 3 simulations that are currently available. Thus justification for the development of Level 4 and Level 5 simulations depends upon their contribution to these two critical areas relative to crash testing. The trade-off discussion that follows will be based on consideration of minimum sophistication required, accuracy and repeatability, accomplishment time, and operational and development costs.

Minimum Sophistication

For the efficient exercise of the functions under consideration, it is desirable to employ a method with the minimum sophistication required to accomplish the task. For example, to predict compliance for a modification in roof structure requires a different level of information than to predict forces imposed on occupants by restraint systems in a frontal impact. Both simulation and crash testing can be conducted over a range of sophistication. In general, however, simulation models have a much broader range of flexibility.

This will be particularly true if the modular development recommended for advanced simulations is employed. In effect this

permits assembling a wide variety of models that can be constructed to focus on the specific task. Crash tests, of course, can vary widely in the form and amount of instrumentation. Nevertheless there is a limitation in flexibility inherent in full scale testing in that the entire vehicle must be employed. This determines the nature of the basic testing facility required independent of the specific task.

Accuracy and Repeatability

In one sense simulation has a clear advantage over crash testing with respect to repeatability of results. A given computer simulation will always produce the same result for a given set of input data. In crash testing the equivalent of input data is subject to experimental variability. It is difficult to reproduce identical conditions from test to test

There are at least two other aspects of repeatability, however, that are less clear. The first is the generation of input data for simulation programs. For the large programs required for Level 5 simulation this is a major effort requiring calculations of structural properties and determination of material behavior. In addition judgmental decisions must be made in discretizing the structure. These factors introduce a degree of variability that is at least equivalent to that encountered in crash testing. Recent advances in automatic computer generation of input data direct from production information will help alleviate but will not eliminate this problem.

The second factor is the considerable variation that exists in nominally identical vehicles. Allowable tolerances, less than ideal quality control, and vehicle degradation all contribute. This problem confronts both simulation and crash testing. It is perhaps more manageable within the context of simulation where it is relatively easy to vary parameters between extremes to bound the variation in computed results. To establish similar bounds through testing is probably prohibitive.

We conclude that both simulation and crash testing require experience, careful error control, and engineering judgment to insure

repeatable results. In addition both must consider vehicle motion in reaching conclusions from a given simulation or test.

The question of accuracy also has several ramifications. We consider two specific factors here.

1. Inherent Accuracy of Method

For computer simulation there is always the question of whether the numerical output is an accurate representation of the solution to the questions, i.e. to the modeling concept. Advances in numerical methods and numerical error control indicate that results with very small error bounds are obtainable for systems of the size contemplated for Level 5 simulation.

The equivalent consideration for crash testing is the accuracy of the methods of data acquisition. The error bounds on physical instrumentation devices have limiting lower values that in general exceed the numerical error bounds. There is also potential sources of error in post processing of data, e.g. in filtering techniques. It can be anticipated, however, that the increasing sophistication of data acquisition systems together with standardization of post processing techniques will insure that experimental data represent the actual event with a satisfactory accuracy. Thus for the purpose of trade-off the level of confidence in numerical accuracy and experimental data acquisition accuracy can be considered as equivalent.

2. Physical Accuracy

A full scale crash test is a real world event. Thus assuming the conclusion in item 1 with respect to the confidence level of data acquisition, the results of a crash test represent the "solution" for that specific event. With respect to simulation programs, the development of Level 4 and Level 5 simulations implies that the accuracy associated with their definition can be achieved. From the discussion above, we conclude that the technical potential exists for this development. Nevertheless a simulation program is only a model of the real world. Thus the crash test must remain the standard by which confidence levels for simulation are measured.

There is, however, another aspect to the question of accuracy. Although a crash test is a real world event, it represents a specific and controlled environment. In the sense that we want to use crash data to predict performance in general environments, the crash test may also be viewed as a model. The confidence level of crash tests decreases as we extrapolate from the specific test conditions. The situation is different for simulations. Once a simulation program has been qualified over its range of applicability, its confidence level is uniform with respect to changes in input over this range. Thus qualified simulation programs which are inherently less accurate than a crash test for specific conditions may predict behavior in a more general environment not amenable to test with a higher level of confidence than the extrapolation of test data.

A similar situation may exist with respect to specific conclusions to be inferred from the simulation or test. An example will serve to illustrate. Suppose we wish to judge compliance with a standard on maximum acceleration of a occupant dummy during frontal barrier impact. A crash test will provide the maximum level of confidence on whether compliance was achieved. If, on the other hand, our purpose was to relate specific structural behavior to achievement of compliance, a simulation result may provide a more accurate conclusion. The model provides results for a wide range of variables that are not amenable to experimental measure. Thus correlation of detailed structural behavior with an observed result may be quantified from simulation data but remains speculative based on crash data only.

We conclude that crash testing must provide the standard of reference in establishing the confidence level of computer simulation. Once established, however, the flexibility of simulation with respect to input conditions and output variables may provide more accurate predictions for the general crash environment than extrapolations of crash test data.

Accomplishment Time

The factors determining accomplishment time for a crash test are vehicle acquisition and preparation, instrumentation set-up time,

testing and data acquisition, and data processing. The comparable items for simulation are input data preparation, computer run-time, and interpretation of output data. In terms of elapsed clock time the computer time is negligible, with the major item being preparation of input data.

Currently accomplishment time for a crash test is less than a simulation program of size comparable to Level 5 simulation. The preparation and checking of input data is a time consuming and tedious task. To a lesser extent handling of output data is a problem, but considerable progress has been made here in the automatic plotting and correlation of computed results. There has also been considerable progress in the development of software to automate the generation of input data for vehicle structures. Thus dramatic reductions in time associated with preparation of input data can be anticipated. It has been estimated [19] that data for programs of Level 5 complexity can be prepared with an expenditure of a few man days.

With respect to crash testing, instrumentation is the major factor currently followed by data processing. Considerable progress has been made in the later category and highly automated experimental data processing can be expected. It is unlikely, however, that instrumentation time can be significantly reduced, and in fact is likely to increase as more information is required. Thus even standardized crash tests will require the expenditure of several days.

We conclude that in the time frame required for the development of a Level 5 simulation it can be expected that accomplishment time for simulation programs and crash tests will be generally comparable. It should be noted that this comparison is based on one run versus one test. In a broader context simulations have a distinct advantage since once the input for a vehicle has been prepared, results for a variety of crash environments can be obtained with nominal accomplishment time, whereas a complete crash test would be required for each event. Thus in this sense simulation has a distinct advantage.

Operational and Development Costs

A comparison of costs of simulation versus crash testing is

difficult because accurate costs are hard to obtain. The cost of a crash test varies widely depending upon the particular vehicle, the nature of the test facility, and the amount of instrumentation required. A minimum value for a full scale test of \$4,000 appears a reasonable estimate with values ranging up to \$20,000 possible. Specialized development tests may run considerably more. With increasing sophistication of standards it can be expected that the cost of compliance testing will also increase.

Estimating costs of advanced simulations is at best speculative. At the present time qualified Level 4 and Level 5 simulations do not exist. Based on our modeling study [16], however, we estimate that the Level 4 simulation employing a modular concept can be exercised for \$200 - \$400 with present generation computers. Some estimate of Level 5 simulation can be obtained from examination of current finite element frame models of vehicle structures. It appears that a factor of ten over Level 4 simulation is reasonable. In addition the preparation of input data is a major item, either by hand or by the cost of exercising software for automated data input. In estimating cost, however, this high initial cost of data input must be reduced due to the fact that a variety of simulations for vehicles with minor differences can be exercised without any significant costs for input. It can also be anticipated that the next generation of computers that will probably be available within the time frame of the development of Level 5 simulations will reduce costs by a factor of 5-10. Thus based on all these considerations we feel that \$2,000 - \$4,000 is a reasonable estimate of the cost of a Level 5 simulation.

It thus appears that computer simulation holds the promise of distinct economic advantage if advanced simulations can be developed. The present trade-off study indicates that simulation has sufficient merit relative to crash testing to justify proceeding if development costs are reasonable. Development costs and the related economic implications are discussed in the next section.

4.5 ECONOMIC CONSIDERATIONS

It is clear from the above discussion that advanced simulations cannot eliminate crash testing. It will always be necessary to experimentally establish base lines of performance with a high confidence level. It is also clear that the flexibility of simulations makes them attractive alternatives to crash testing. Moreover once experimental base lines are established, advanced simulations could significantly reduce the required level of crash testing. Nevertheless to justify a recommendation to develop advanced simulations requires demonstration that development costs are not prohibitive.

Our estimates of development costs are primarily based on the modeling study conducted under this contract. We estimate that a qualified Level 4 simulation can be developed in two years with an annual expenditure of two hundred to two hundred fifty thousand dollars. Development of a qualified Level 5 simulation will require a somewhat longer time frame with five years being a reasonable estimate. During the first two years the two efforts will be mutually supportive since a number of questions, e. g. joint behavior, are relevant to both modeling efforts. Expenditures for the Level 5 effort can also be anticipated to average two hundred thousand per year. Thus we conclude that the spectrum of advanced simulations can be developed for approximately 1-1/2 million dollars over a five year time frame.

This development cost must be projected against potential savings. At the present time annual compliance testing is on the order to fifty vehicles per year. Pressure for higher confidence levels is likely to increase this to order of one hundred and fifty vehicles per year in the near future. (Potential developments under the Motor Vehicle and Cost Saving Act could increase this substantially.) It is likely that an equivalent number of tests are conducted by industry. It is also reasonable to assume that the number of full scale development tests conducted will be on the same order of magnitude as compliance testing. Thus in the near future 5-6 hundred full scale tests will be conducted annually. Even at an average of \$6,000 per test this represents an annual investment of 3.0 - 3.6 million dollars.

As discussed above advanced simulations will not and should not eliminate crash testing. In our opinion, however, the existence of advanced simulation could reduce crash testing by 75%. Moreover with a high confidence level base line established by crash tests, we believe that Level 4 simulation would suffice for one-half of all tests replaced.

Based on this premise the annual savings of employing simulation are shown in Figure 1. The solid curves are based on an average simulation cost of \$1,650 (Level 4 at \$300 and Level 5 at \$3,000). They show savings as a function of number of total tests and the cost per test. The dash lines show savings as a function of number of tests and cost per simulation based on an average crash test cost of \$6,000. The Figure shows that for a wide range of level of testing and associated costs, development costs would be recovered in a single year. Moreover for almost any reasonable estimate of these variables, development costs would be recovered in two years.

These rather striking results are, of course, a function of the assumption on reduction in crash testing. But even if simulation (at an average cost of \$1,650) reduced crash testing (at an average cost of \$6,000) by 25%, development costs would be recovered in three years if annual testing was at the level of 400 vehicles. Thus the development of advanced simulations has the potential for substantial economic benefits.

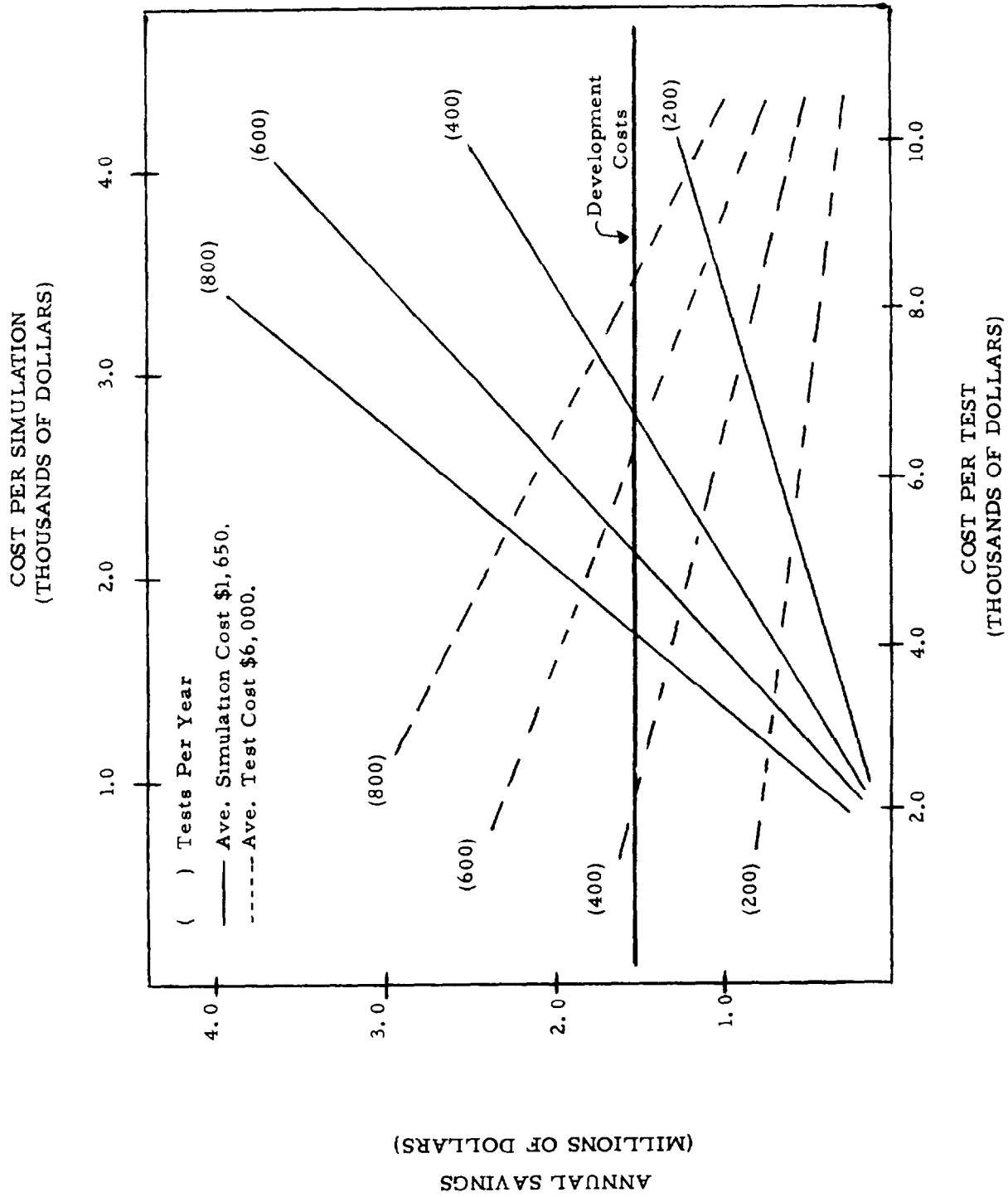


FIGURE 1 PROJECTED SAVINGS FOR USE OF SIMULATION

CHAPTER 5

RECOMMENDATIONS FOR DEVELOPMENT OF ADVANCED SIMULATIONS

5.1 REQUIREMENTS FOR ADVANCED SIMULATIONS

Advanced simulations must have the capability of predicting the structural response of vehicles under a variety of impact loadings. Moreover they must be flexible, allowing a variety of modeling concepts to be integrated into a simulation appropriate for a particular loading situation. Thus advanced simulations must meet the following general specifications:

- 1) Deformation variables must allow three dimensional displacements and rotations valid for large deformations.
- 2) Permissible loading configurations must include barrier, pole, and vehicle-to-vehicle impact in unsymmetric and oblique configurations involving frontal, side, and rear collisions.
- 3) The simulation program must permit the automatic generation of the system equations for user specified arrangement and number of component modules.

With respect to specific levels of simulation, Level 4 simulation should meet the following specifications:

- 1) The code formulation should be such as to compute the following items:
 - a) Energy absorbed by various structural components and total energy dissipated by the structure.
 - b) Relative displacements of major components.
 - c) Acceleration environment of passenger compartment.
 - d) Intrusion of external obstacles or major components into passenger compartment.
- 2) Computed results should have an accuracy comparable with testing.
- 3) The total simulation should employ on the order of 300 degrees of freedom.

Level 5 simulations should meet the following specifications:

- 1) The code formulation should compute the displacement and acceleration time histories of all significant points in the vehicle. In particular both the deformation and acceleration of the occupant compartment should be determined in detail.
- 2) Computed results should have an accuracy comparable to testing.
- 3) The total simulation should employ less than 2000 degrees of freedom.

5.2 CONCEPT OF MODULAR DEVELOPMENT

As discussed in Chapter 4, it is our opinion that the technical potential exists for the development of advanced simulations. Nevertheless this development offers a major challenge to the crashworthiness effort. To realize the potential economic benefits the size restrictions cited in the previous section must be imposed. This implies specialized modeling techniques designed for the vehicle impact problem based on experience with testing. On the other hand, the requirements for advanced simulations require sufficient generality to treat a wide variety of loads and structural response.

We recommend that the approach to advanced simulations be based on the development of a number of self-contained mechanical simulation modules representing sub-assemblies of the vehicle. Some modules would be general purpose like a frame module or rigid body module. Others would represent specific sub-assemblies like a drive train or suspension module. Even for advanced simulations a module which can be defined by empirical test data is likely to be required.

For each module we define a discrete set of "external nodes" as the points where it interacts with other modules. Enforcing compatibility and dynamic equilibrium of the nodes gives the overall system equations. This is identical to assembling the global equations in a finite element method where the simulation modules are analogous to "super elements".

This modular development has a number of advantages. It permits employment of a variety of modeling techniques that are appropriate for

specific components. It also permits freedom within one program for assembling quite different models for particular situations, thus minimizing the number of degrees of freedom employed. This is controlled by input data specifying the numbering and initial location of the external nodes of the appropriate modules chosen from a module library. The program system will then generate the equations governing the overall response. The organization of input data would be simplified, each module having its own format compatible with the modeling technique employed.

In developing the individual modules, the choice of modeling technique would be based on the most efficient method compatible with the detail and accuracy required for a specific component relative to its role in the overall vehicle response. Level 4 and Level 5 simulations would be accomplished within the same framework, the only difference being in the number and modeling sophistication of the modules employed. For example a frame module based on the plastic hinge concept might be employed in Level 4 simulation, whereas a finite element frame module would be required for Level 5.

To effect this modular approach will require an intensive effort directed towards component definition and modeling. There are three basic steps that must be accomplished. They are:

- 1) Identify vehicle sub-assemblies and determine appropriate modeling concepts for each component.
- 2) Develop self-contained simulation modules for each modeling concept.
- 3) Develop a computer executive system for assembling individual module simulations under control of input variables.

Table 4 indicates a number of vehicle sub-assemblies that would need to be considered. A tentative identification of the type of module required is shown. The role of joints in the structural response is sufficiently important to list them as a sub-assembly. Most joints can be probably treated by a general joint program that might be incorporated within the general purpose modules. On the other hand special modules will undoubtedly be needed for such connections as motor mounts and steering mechanisms attachments.

SUB-ASSEMBLY	TYPE OF MODULE
Frame Structure	General Purpose
Unitized Body Structure	General Purpose
Exterior Sheet Metal	General Purpose
Rigid Components (Motor, Transmission)	General Purpose
Mechanical Assemblages (Drive Train, Steering Mechanism)	Special
Forestructure (Bumper, Grill, Radiator)	Special
Suspension	Special
Passenger Compartment	Variable
Joints	Both General and Special

TABLE 4: VEHICLE SUB-ASSEMBLIES

It should also be noted that most sub-assemblies may be modeled by different modules depending upon the specific simulation. For example, a plastic hinge frame module or a finite element frame module might both be used to model a frame structure depending upon the level of simulation. Finally the passenger compartment is classed as a sub-assembly due to its paramount importance. It may actually be modeled, however, by a combination of other general purpose modules and specialized modules for specific parts of the compartment like doors or interior structure. The passenger compartment is a good illustration of the flexibility of the modular approach. Table 5 shows possible combinations of modules to define a passenger compartment module for various simulation conditions.

SIMULATION CONDITION	MODELING CONCEPT
Primarily Frontal Level 4	Three Dimensional Rigid Body with Defined Space Enveloped
Side Impact Level 4	Rigid Body Elements, Plastic Hinge Frame, Equivalent Beam Door Module
Primarily Frontal Level 5	Plastic Hinge Frame Module
Side Impact Level 5	Finite Element Frame, Detailed Door Module

TABLE 5: PASSENGER COMPARTMENT MODULES
FOR VARIOUS SIMULATION CONDITIONS

5.3 DEFINITION OF REQUIRED MODULES

At the present time, modeling concepts required for all vehicle sub-assemblies are not well defined. Nevertheless the basic modules required and potential approaches to their development can be identified. The required modules can be grouped in three main categories. They are:

- 1) Control Modules
 - a) Executive System
 - b) Input - Output System
- 2) General Purpose Modules
 - a) Level 4 Frame
 - b) Level 5 Frame
 - c) Two Dimensional Level 4 Structure
 - d) Two Dimensional Level 5 Structure
 - e) Three Dimensional Rigid Body
- 3) Specialized Modules
 - a) Joint Module
 - b) Door Module
 - c) Bumper
 - d) Grill-Radiator
 - e) Mechanical Assemblages
(Drive train, steering, suspension)

A number of desired features for the control modules are summarized in Table 6. Although the development of the control modules is a major task, it does not require any new computer technology. For example, the ability to define space envelopes and to add interactions as the program proceeds are features of some currently available simulations.

CONTROL MODULE	DESIRED CAPABILITIES
EXECUTIVE SYSTEM	<ol style="list-style-type: none">1. Assemble System Equations from User Specification of Modules and Location of Nodes Defining Module Interactions.2. Define and Monitor Space Envelopes Associated with Selected Elements.3. Ability to Add Nodal Interactions by Monitoring Intrusion of Space Envelopes.
INPUT SYSTEM	<ol style="list-style-type: none">1. Designed to Handle Input on Modular Basis.2. Automate Computation of Input Data for Nodal and Element Geometry for Selected Modules.
OUTPUT SYSTEM	<ol style="list-style-type: none">1. Provide Flexible Output Format to Print Out Results for User Specified Variables.2. Provide Graphical Display for User Specified Variables.

TABLE 6: REQUIREMENTS FOR CONTROL MODULES

Some work will be required to develop the dynamic equilibrium and compatibility conditions at the nodes where modules with different modeling concepts interact. The general approach to this problem, however, is well documented in the finite element literature.

The basic modeling concepts for the general purpose modules are also well advanced. A Level 4 frame module has been developed in its general features under this contract and is described in Volume 4. The finite element frame programs discussed in Chapter 3 provide a strong foundation for the development of a Level 5 frame program. A three dimensional rigid body module can easily be programmed. Two dimensional structural modules are less clear. A Level 5 module is probably technically

feasible within current finite element technology. (For a general discussion and other references see references [20] [21]). It may be necessary, however, to develop rather specialized elements for vehicle sheet metal. The most difficult problem is appropriate modeling concepts for a two-dimensional Level 4 module. Such a module will play a major role in Level 4 simulations and is also important for Level 5 simulations where it will be needed to efficiently model less critical regions of the vehicle. Extensions of the plastic hinge concept to hinge lines or the rational definition of an equivalent beam are possible approaches. At the present time, however, no progress has been made in this direction.

Modeling concepts for the specialized modules are not well developed. Some work is available on modeling the door structure and bumper systems. Major efforts have been made for modeling suspension systems but not from the viewpoint of vehicle impact. Suspension models relevant to the impact problem will need to be developed. For the most part, however, the development of such modules through mechanical simulation should be relatively straight forward once their role in the vehicle impact problem is identified. This work will necessarily be based on component testing. The major problem area is joint behavior. The effect of joints on the structural response is significant, but there is no method currently available for incorporating these effects in the large plastic deformation range.

The various required modules are summarized in Table 7. Appropriate modeling concepts are indicated. A question mark is indicated if the potential of the modeling concept is not well established. Approximate size restrictions on the modules are indicated if they are to be feasible as components of an overall simulation. Of course, considerable flexibility is possible in a given module depending upon their use relative to other modules in the particular overall simulation. Finally, development priority is indicated. These priorities are based on first developing Level 4 simulation capability. Since most of the modules required for this level will also be required in Level 5 simulations, priority for Level 4 development is reasonable.

MODULE	MODELING CONCEPT	USE	MAXIMUM SIZE	DEVELOPMENT PRIORITY
LEVEL 4 FRAME	PLASTIC HINGE	MAJOR STRUCTURE - LEVEL 4, MINOR STRUCTURAL ELEMENTS - LEVEL 5	150	HIGH
LEVEL 5 FRAME	ELASTIC-PLASTIC FINITE ELEMENTS	MAJOR STRUCTURE LEVEL 5	600	LONG RANGE
LEVEL 4 TWO-DIMENSIONAL STRUCTURE	PLASTIC HINGE LINE? EQUIVALENT BEAM?	PRIMARY SHEET METAL OR UNITIZED CONSTRUCTION - LEVEL 4, MINOR SHEET METAL - LEVEL 5	75	HIGH
LEVEL 5 TWO-DIMENSIONAL STRUCTURE	ELASTIC-PLASTIC FINITE ELEMENTS	UNITIZED CONSTRUCTION, PRIMARY SHEET METAL - LEVEL 5	600	LONG RANGE
RIGID BODY	RIGID BODY DYNAMICS	GENERAL USAGE	6/RIGID BODY	HIGH
JOINT	?	GENERAL USAGE	LIMITED	HIGH
DOOR	EQUIVALENT BEAM BEAM-MEMBRANE	LEVEL 4 LEVEL 5	6 30	INTERMEDIATE
BUMPER	VARIOUS-DEPENDING UPON DESIGN	GENERAL USAGE	10-30	INTERMEDIATE
GRILL-RADIATOR	EMPIRICAL?	GENERAL USAGE	6	LOW
MECHANICAL ASSEMBLAGES	GENERALIZED SPRING-MASS	GENERAL USAGE	6-18 EACH ASSEMBLAGE	INTERMEDIATE

TABLE 7: MODULES REQUIRED FOR ADVANCED SIMULATIONS

5.4 RESEARCH NEEDS

In addition to developing component modules, we can identify a number of basic topics that require investigation in support of the modeling effort. It is highly unlikely that advanced simulations can be realized without consideration of these areas. They are in order of our assessment of priority:

1) Joint Behavior

A major factor in the geometric complexity of automotive structures is the complicated joints and material attachments used in standard manufacturing practice. There is a need for a systematic study of joint behavior under various load conditions. At the present time there even remain basic questions on how to characterize joint behavior. For example, the concept of joint efficiency introduced for elastic joints is not well defined for plastically deforming joints.

2) Local Deformation

For the large deformations experienced in crashworthiness applications there are significant changes in cross sectional shape of structural frame members. It is conceptually possible to model this behavior with three-dimensional finite elements. In practice, however, this is likely to add a prohibitive number of degrees of freedom. Moreover we are not interested in the details of the local deformation but only its effect on the overall load transmission and energy absorbing characteristics of the structure. A rational way to incorporate these effects is needed.

3) Load Transmission Characteristics

For Level 4 and the minor components of Level 5 simulations, restrictions on the total degrees of freedom prohibit using two and three-dimensional finite elements for modeling all non-frame members. Thus there is a strong need for understanding how two-dimensional structural elements transmit various loadings in order to rationally define an equivalent frame member.

4) Strain Rate Sensitivity

Although considerable information is known about material strain rate effects, the realistic incorporation of such effects in structural theories is not well understood. At the present time most simulations use an average strain rate either as an empirical correction factor or to choose a single dynamic stress-strain curve. In general, however, there are large spatial variations in strain rate throughout the structure. The effect of such variations is not known.

5) Numerical Error Control

Numerical methods employed in current simulation programs are generally adequate. There are, however, a number of areas of improvement important to advanced simulations. Current methods in general require considerable judgment and numerical experiments to choose a time step and/or error measure. There is a strong need for systematic study of the effect of local error bounds on accuracy and efficiency. Related questions are the appropriate definition for the error measure and the choice of error weight functions.

REFERENCES

1. McIvor, I. K., "Modeling and Simulation as Applied to Vehicle Structures and Exteriors", NHTSA Research Symposium, May, 1973.
2. Emori, R. I., "Analytical Approach to Automobile Collision", SAE Paper 680016, January, 1968.
3. Tanii, M. and Emori, R. I., "A Study on Automobile Crashworthiness", SAE Paper 700175, January, 1970.
4. Miura, N. and Kawamura, K., "An Analysis of Deformation Mechanisms in Head-On Collision", SAE Paper 680484, May, 1968.
5. Carter, R. L., "Passive Protection at 50 Miles per Hour", U. S. Department of Transportation, Report No. DOT-HS-810-197, 1972.
6. Spencer, P. R., "Parametric Study of Side Impact Vehicle Agressiveness", 18th Annual Meeting of Institute of Environmental Sciences, May 2, 1972.
7. Herridge, J. T. and Mitchell, R. K., "Development of a Computer Simulation Program for Collinear Car/Car and Car/Barrier Collisions", BCL Report DOT-HS-800-645, January, 1972.
8. Herridge, J. T. and Mitchell, R. K., "Initial Crash Test Screening Studies Utilizing The NHTSA/BCL Computer Simulation Program ...", Battelle Columbus Laboratories, Final Report, DOT-HS-800-730, August, 1972.
9. Kamal, M. M., "Analysis and Simulation of Vehicle to Barrier Impact", SAE Paper 70044, 1970.
10. Autosafety Engineering Corporation, "Crash Environment Systems Studies", undated.
11. Shieh, R. C., "Basic Research in Crashworthiness II - Large Deflection Dynamic Analysis of Plane Elasto-Plastic Frame Structures...", CAL Report No. YB-2987-V-7.
12. Wittlin, G. and Gamon, M., "Experimental Program for Developing Rotary Wing Aircraft Structural Crashworthiness Criteria and Concepts", Lockheed-California Co., Final Report, Contract No. DAAJ02-71-C-0066.

13. Young, J. W., "CRASH. A Computer Simulation of Nonlinear Transient Response of Structures", Flalco-Ford Report DOT-HS-09-1-125B, March, 1972.
14. Melosh, R. J., "Car-Barrier Impact Response of a Computer Simulated Mustang", Flalco-Ford Corporation, Report No. DOT-HS-091-1-125A, 1972.
15. Thompson, J. E., "Vehicle Crash Predictions Using Finite Element Techniques", SAE Paper No. 730157, January, 1973.
16. McIvor, I. K., Wineman, A. S., Anderson, W. J. and Wang, H. C., "Modeling, Simulation, and Verification of Impact Dynamics, Volume 4: Three Dimensional Plastic Hinge Frame Simulation Module", Highway Safety Research Institute Report No. UM-HSRI-BI-73-4-4, University of Michigan, August, 1973.
17. Oden, J. T., Cough, R. W. and Yamamoto, Y., "Advances in Computational Methods in Structural Mechanics and Design", University of Alabama Press, 1972.
18. Ni, C. M., "Impact Response of Curved Box Beam-Columns with Large Global and Local Deformations", AIAA Paper No. 73-401, 14th Structures, Structural Dynamics, and Materials Conference, March, 1973.
19. Peterson, W., "Application of the Finite Element Method to Predict Static Response of Automotive Body Structures", SAE Paper 710263, Automotive Engineering Congress, Detroit, 1971.
20. Zienkiewicz, O. C., "The Finite Element Method in Engineering Science", McGraw Hill, 1971.
21. Oden, J. T., "Finite Elements of Nonlinear Continua", McGraw Hill, 1972.